Permutations, Combinations, & The Fundamental Counting Principle

Odds & Probability

By

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Standards:

5.12.5 Determine the probability of an event with and without replacement using sample spaces. Design, conduct, analyze, and effectively communicate the results of multi-stage probability experiments.

5.8.5 Differentiate between the probability of an event and the odds of an event.

5.12.4 Apply permutations and combinations to mathematical and practical situations, including the Fundamental Counting Principle.

5.8.4 Find the number of combinations possibly in mathematical and practical situations. Distinguish between permutations and combinations.

Essential Understandings for Permutations, Combinations, and Fundamental Counting Principle:

Can you determine the number of possibilities that a probability experiment could produce using tree diagrams and/or the Fundamental Counting Principle?

Can you find the number of permutations possible in a given experiment?

Can you find the number of combinations possible in a given experiment?

Can you determine when to use permutations and when to use combinations?

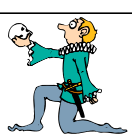
Essential Understandings for Odds & Probability:

Can you compute the odds on a given probability experiment?

Can you tell the difference between odds and probability?

Can you determine from the probability and odds from the results of an experiment? From your results, can you explain how the experimental probability differs from the theoretical probability and why that would occur?

<table>
<tr><td colspan="2">1ST PLACE</td><td>2ND PLACE</td></tr>
<tr><td>SHANE </td><td>HOLLY </td><td rowspan="2">POSSIBILITIES:</td></tr>
<tr><td>Carson </td><td>LESLIE </td></tr>
</table>

<table>
<tr><td colspan="3">Team of 2</td></tr>
<tr><td>SHANE </td><td>HOLLY </td><td rowspan="2">POSSIBILITIES:</td></tr>
<tr><td>Carson </td><td>LESLIE </td></tr>
</table>

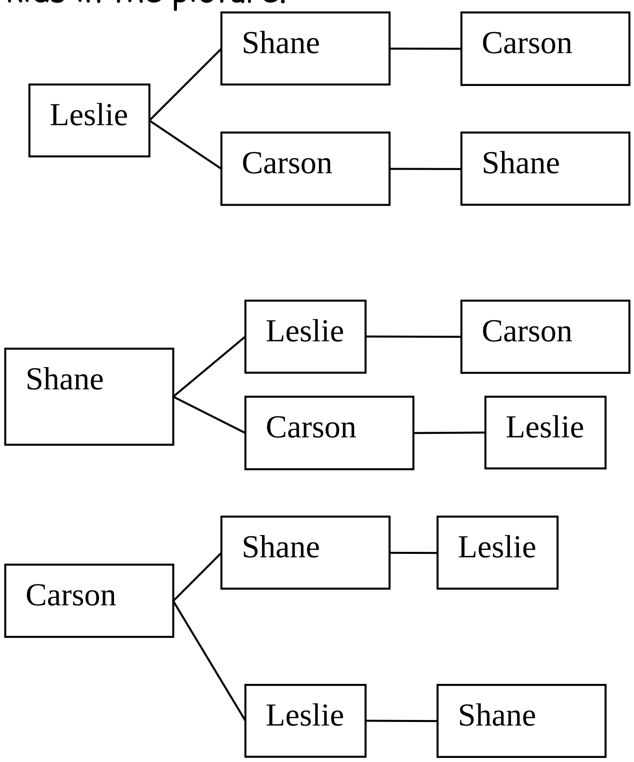
How many different pictures can you take of the three people if they all have to be in each picture? Write out all the possibilities.

LESLIE 	SHANE 	CARSON 

Lesson Notes for Permutations, Fundamental Counting Principle, Combinations

1) Have students do the first sorting/solving activity. How many different pictures can you take of the three people if they all have to be in each picture? Write out all the possibilities.

- Have the students complete the picture table. They should get 6 as the answer. Ask them, what would you do if you added another person to the picture? The answer would get much larger – 24. Writing out all these possibilities isn't always reasonable, but let's make a tree diagram for the scenario where there are 3 kids in the picture.



3 choices for the first person selected, 2 choices left for the second person, and 1 choice for the final person selected. What we want to show kids is WHY they would multiply together. I have 3 sets, then for each of those I have two sets, then for each of those I would have 1 set.

2) Read <u>Anno's Mysterious Multiplying Jar</u> by Mitsumasa Anno. This introduces the concept of factorials as well as reiterates the concepts of "for each I have so many sets of." So, if we go back to the pictures example and have 4 students to pick from, then it would look like this ____4____ x ____3____ x ____2____ x ____1____ = 24 or 4!

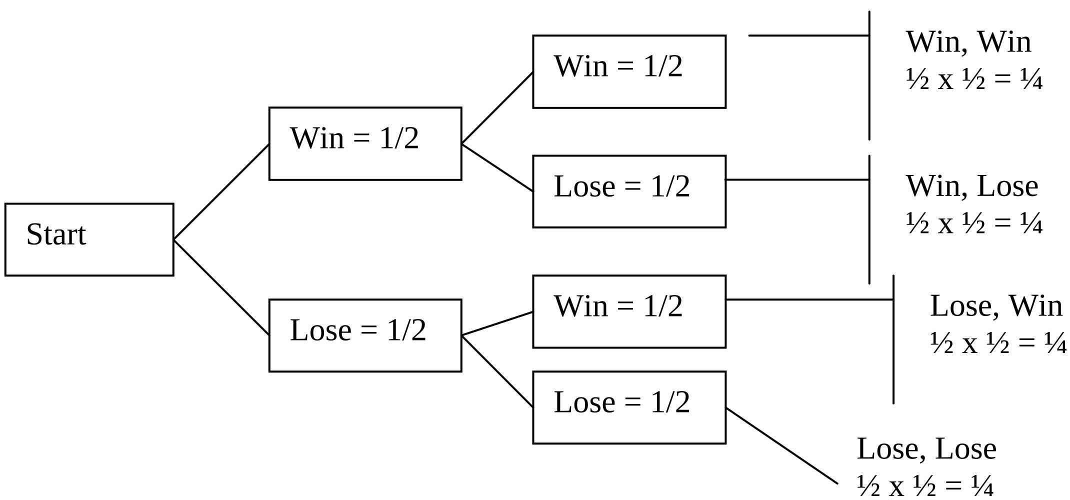
3) Give a couple of example problems:

a) You have 6 books to arrange on a shelf, in how many different ways can you arrange all the books? Answer – 6! Or 720

b) There are 5 prizes to be given out to five people, how many different ways can people be awarded prizes. Answer – 5! Or 120

4) Let's extend the Fundamental Counting Principle to help us calculate probability on repeated events. This helps students understand better why we have to MULTIPLY rather than add when we are calculating repeated events in probability.

Example: You are playing a game (flipping a coin) twice, where the chances of winning each time you play are independent and calculate to ½. That means the probability of losing is also ½. Let's make a tree diagram and see how it can help us match up what we know the sample space to be { (Win, Win), (Win, Lose), (Lose, Win), (Win, Win)}



When you move along the path of the tree, you have to multiply, because the sample space is getting big by the fundamental counting principle.

5) Try a few examples:

a) You are playing a game where the probability of winning is 1/3. If you play the game four times, what is the probability that you will lose all four times? Answer – 1/81.

b) A spinner has 8 equal parts, labeled A – H. If you spin the spinner twice, what is the probability it lands on C two times in a row? Answer – 1/64.

6) Extensions of the Fundamental Counting Principle – How many phone numbers can I have if I have 7 digits in the phone number? 10 x 10 x 10 x 10 x 10 x 10 x 10. You can discuss that phone numbers don't start with zero, so it might be better to do 9 x 10 x 10…

a) How many license plates can be made if there are 3 numbers and 3 letters?

Answer – 10 x 10 x 10 x 26 x 26 x26

b) How many license plates can be made if there are 3 numbers and 3 letters and no letter or number may repeat? 10 x 9 x 8 x 26 x 25 x 24

7) **Permutations** – what if you want to find the number of ways that you can take a group of objects from a total set? Activity – have the students put in the people in the first and 2nd place race and see how many possibilities that they get. It is best to start with the question, how many ways can I award 1st and 2nd place to 4 people? The reaction that students may initially have is 4! Or 24. So, let them place the people and record the answers. They should get 12 possibilities.

Ask the students to use the same pieces, but answer, what if there was 1st, 2nd, and 3rd place to award for those 4 people? Now there are 24 possibilities.

What if there is only 1st place? Now there are only 4 possibilities.

When you take sub-groups from an original group the possibilities change depending on the size of the sub-group. This does use the Fundamental Counting principle, but it extends it to something called a **permutation**. The formula for Permutation is defined to be when we have a total of n objects and we are taking a sub-group of r,

$\frac{n!}{(n-r)!}$ or it is also written as ${}_nP_r$. See if you can find it in their calculator or do the examples above using the formula, showing how factorial written out can cancel.

If there are 4 people and you need to pick a 1st, and 2nd place winner, how many ways that be done? N=4 and r=2 so $\frac{4!}{(4-2)!} = \frac{4 \bullet 3 \bullet 2 \bullet 1}{2 \bullet 1} = 4 \bullet 3 = 12$

Have the students calculate 4 people with 1st, 2nd, and 3rd place winners, 4 people and 1st place winner only, and 4 people and 1st, 2nd, 3rd, and 4th.

8) Do some example problems.

a) If you have 5 people, how many different photographs can you take with only 4 people in the picture? Answer: 120

b) You have 6 people in your club, how many ways can you choose President, Vice-President, and Secretary? Answer: 120

c) There are 5 colors to choose from, how many unique 3 color flags can be made? Answer – 60

9) **Combinations** – Have students put the 4 people in teams of 2. Start with asking the question, how many ways can we take four people and put them in teams of 2? They many do a permutation and answer 12. Have them work the people and record the possibilities to see. Make sure to point out as they are working isn't a team of Shane and Carson the same thing as a team of Carson and Shane?

With those same people, have them calculate a team of 1 and then a team of 3? Have them compare their results to those when we did 1st, 2nd, and 3rd place. Why is this different? They need to be guided to the fact that ORDER doesn't matter (within the sub-group) for these types of problems, where ORDER did matter for permutations and fundamental counting principle problems.

Combinations – n is the total group and r is the number in the sub-group: $\dfrac{n!}{(n-r)!r!}$

The other way it is written is ${}_nC_r$. See if students can find it on their calculator. Now, do the calculations with the above finding a team problem using the formula. Focus on canceling out the factorials and doing order of operations properly.

10) Let's do some example problems:

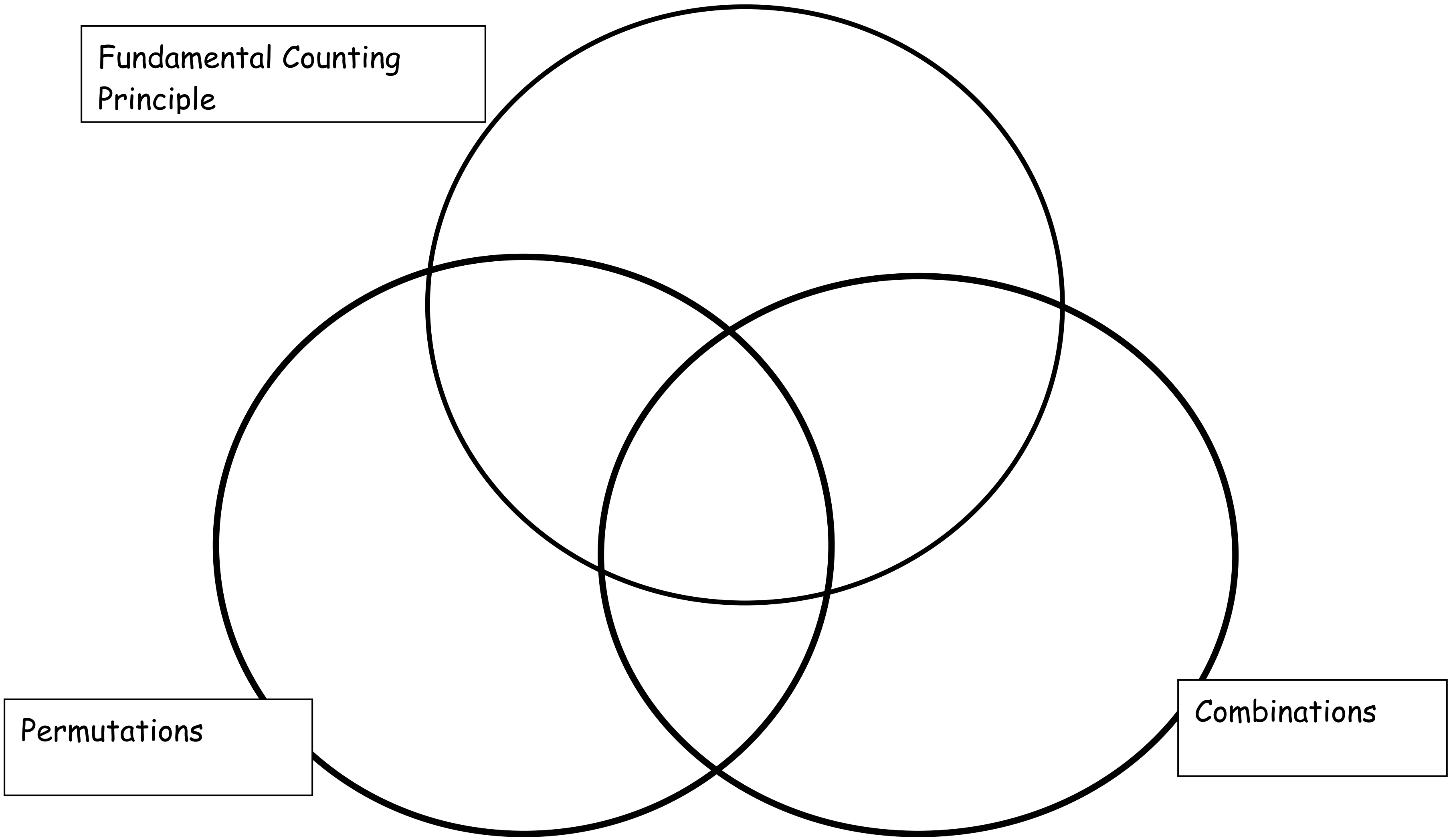
a) You need to pick three people to be on a committee. There are 7 people to pick from, how many different committees can you make? Answer - 35

b) A four man bowling team is chosen from 5 people, how many different teams are there? Answer – 5

11) In order to stress the difference between permutations, combinations, and fundamental counting principle, have the students fill out the tri-venn diagram with a partner. Go over as a class. Make sure that they are looking out for things like ORDER and sub-groups. Writing Extension: From their completed tri-venn diagrams, have the students write a haiku (5 words, 7 words, 5 words)

12) Choices for in-class cooperative work or homework:

- Matrix – representing problems in a different way
- Homework handout (same problems from the matrix above)
- Constructed response (teacher assess, or students may assess)
- Additional vocabulary development on odds vocabulary matrix


Fundamental Counting Principle
Permutations
Combinations

Essential Understandings: Can you apply a permutation or a combination to a practical situation? Can you determine when to use a permutation, combination, or the fundamental counting principal? (5.12.4, 5.8.4)

For each problem, you must complete the shaded columns. After that you may choose 2 columns to complete for each problem.

Problem	I know this is either a permutation/combination/ or Fundamental Counting problem because:	Calculate the answer showing all the work.	Draw a tree diagram or picture representation.	Write a list of all the possibilities, for example for tossing a coin twice would be {(H,H), (H, T), (T,H), (T,T)}	Explain how this problem is different than the one above or below it (you may not say because the answer is different or that the numbers used are different, you need to think deeper about the type of problem).	Write a new problem with a partner that uses the same solving method and give the solution. (You may not re-write the same problem with the same numbers)
You need to select a 3 person dance committee from 5 person group.						
From a group of 4 girls, we must select a homecoming queen.						
How many different						

ways can 4 senators sit on a committee panel?						
How many ways can 3 salad dressings be shown in a line in a display case?						
Five people entered a baking contest, how many different ways can we give 1st prize?						
How many 3 topping pizzas can you make from 5 choices?						

1) Introduce **Odds** – the number of favorable outcomes/number of unfavorable outcomes
    a. Define favorable & unfavorable – relate it to sample space and the calculation of probability.
    b. Do a **character representation** of favorable and unfavorable. *Your birthday is coming and that means a family party with the relatives. Your favorite Aunt is invited as well as your least favorite person Uncle Frank. Your favorite Aunt always gives you everything you want. If you give a list of possible choices for your family to choose from she always somehow knows which one you really want the most and gets it for you every time. She dresses in fun clothes, the kind that you really like. Your Uncle Frank, on the other hand, dresses in clothes you don't like. Uncle Frank looks at your list of possible choices and somehow finds the ones that your Mom put on there for you – like underwear, toys too young for you, or movies that you grew out of years ago. You have never wanted a single gift that he has given you, but you can't be rude and ignore him.* You might want to have students draw a picture of the favorite Aunt and Uncle Frank, then label them favorable and unfavorable. How does their character representation relate to their definitions?
    c. Do some examples together:
        i. A spinner has 6 equal parts numbered 1-6. If you spin the spinner once, what is the probability of landing on an even number? Answer 3/6 or ½.
        ii. What are the odds of getting an even number on one spin? 3:3 or 1:1
        iii. What is the probability of getting a number less than or equal to 2, $P(x \le 2)$? Answer: 2/6 or 1/3
        iv. What are the odds of getting a number less than or equal to 2? Answer: 2:4 or 1:2.
2) **Activity:** Looking at Casino games, students will have a chance to checks the odds given on a game (or a modification of a game). The students can choose which experiment they want to test, but each member will fill out their own paper. Have groups report back on their findings either by putting their results and discussion on chart paper, or by explaining back to the class. #1 medium, #2 hard, #3 easier
3) **Activity:** Theoretical Odds and Probability Bingo: Have students take the squares, cut them out, and glue 16 in the blank bingo card box. The teacher will need to make an overhead of the questions, place masking tape under the answers, cut apart the questions and shuffle.
4) **Homework or further activities**:
    - Task rotation
    - Vocabulary matrix reviewing many topics
    - Theoretical Bingo passed out with the questions for the students to go over and complete at home (they have the answers from their cards to check).
    - Constructed response.

Experiment #1: Losing at Craps

Explanation: When playing craps, for the most part you lose if a player rolls a 7 or 11.

Experiment: Roll 2 dice and total the numbers on the face. Example, if you roll a 4 on one die and a 5 on the other die, you have rolled a 9 and that is what is recorded. You will roll 2 dice 50 times and record the total of the two dice.

Record your results from the 50 rolls here:

Now, organize your results in some way:

What is your sample space? _______________ (hint: think total)
How many times did you lose in your experiment (favorable outcomes)? ______________

What is the experimental probability that you will lose? ________________
What are the odds that you will lose in your experiment? _________________

Let's examine Theoretical probability (what should happen). Write out the possible combinations from rolling two dice in the table below. It has been started for you:

one dice ⇒ other dice ⇓	1	2	3	4	5	6
1	(1,1)=2	(2,1)=3				
2						
3						
4						
5						
6					(5,6)=11	

Looking at your chart at the possible outcomes, how many outcomes are in the sample space? _______________
How many outcomes force the person to lose (favorable outcome)? ___________

What is the theoretical probability of losing? _____________
What are the odds of losing? ______________

How do your experimental results for the probability of losing and odds of losing differ from the theoretical results?

Explain what would cause those differences to occur?

Do you and your group think this is a fair game? Explain why you think that by using your results to your experiment or the theoretical probabilities:

Experiment #2: Winning at Craps

Explanation: When playing craps, for the most part you can win big if a player rolls a double 4.

Experiment: You will roll 2 dice 50 times and record the outcome of the two dice.

Record your results from the 50 rolls here:

Now, organize your results in some way:

What is your sample space? _______________ (hint: think total)

How many times did you win in your experiment (favorable outcomes)? ______________

What is the experimental probability that you will win? ________________

What are the odds that you will win in your experiment? _________________

Let's examine Theoretical probability (what should happen). Write out the possible combinations from rolling two dice in the table below. It has been started for you:

one dice ⇨ other dice ⇩	1	2	3	4	5	6
1	(1,1)	(2,1)				
2						
3						
4						
5						
6					(5,6)	

Looking at your chart at the possible outcomes, how many outcomes are in the sample space? _______________

How many outcomes allow the person to win (favorable outcome)? __________

What is the theoretical probability of winning? ____________
What are the odds of winning? _____________

How do your experimental results for the probability of winning and odds of winning differ from the theoretical results?

Explain what would cause those differences to occur?

If you play craps and bet on double 4's and you win, you get your dollar back and $ 6 more. Do you and your group think this is a fair game? Explain why you think that by using your results to your experiment or the theoretical probabilities:

Experiment #3: Winning at Roulette

Explanation: When playing roulette, you can bet on black or red and if it comes up, then you win.
Experiment: Flip a coin fifty times making reds = heads and black = tails. Let's say that you are betting on red to win.

Record you results from the 50 rolls here:

Now, organize your results in some way:

What is your sample space? _______________ (hint: think total)
How many times did you win in your experiment (favorable outcomes)? ______________

What is the experimental probability that you will win? ________________
What are the odds that you will win in your experiment? _________________

Let's examine theoretical probability (what should happen) for flipping a coin. What are the possible outcomes (sample space) for flipping a coin once? ___________

Looking at your possible outcomes, how many outcomes are in the sample space? _______________
How many outcomes force the person to win (favorable outcome)? ___________

What is the theoretical probability of winning? _____________
What are the odds of winning? ______________

How do your experimental results for the probability of winning and odds of winning differ from the theoretical results?

Explain what would cause those differences to occur?

The payout for this game is if you bet one dollar on red and it comes up red, then you get your dollar back plus one more dollar. Do you and your group think this is a fair game? Explain why you think that by using your results to your experiment or the theoretical probabilities:

Constructed Response:

An experiment where there are 10 numbered balls in a bucket. A person draws a number, records it, then puts it back. The results from the experiment are listed below:

Trial	1	2	3	4	5	6	7	8	9	10
Number drawn	6	9	2	8	9	10	6	3	4	1

A) What are the odds of drawing an even number from this experiment?
B) How does the experimental probability of drawing an even number compare with the theoretical probability?
C) Discuss what could account for the difference between the two types of probability – theoretical and experimental.

Your answer will be scored primarily on the following rubric.

Score	Expectations
Full Credit **3**	• Your response addresses all parts of the question clearly and correctly. You use and label the proper math terms in your answer. • Your response shows all the steps you took to solve the problem.
Partial Credit **2**	• Your response addresses most parts of the question correctly. • Your response does not show all of your work or does not completely explain the steps you took to solve the problem.
Minimal Credit **1**	• Your response addresses only one part of the question correctly and explains the steps you took to solve that one part. In answering the remaining parts of the question, your response is incomplete or incorrect. • Your response does not show all of your work or does not explain all the steps you took to solve the problem.
No Credit **0**	• Your response is incorrect. • You did not attempt the problem.

<table>
<tr><th colspan="2">Constructed Response Scoring Rubric for Fundamental Counting Principle and Permutations</th></tr>
<tr><td>3</td><td>(Full Credit)

A) (1pt) 6/10 or 3/5 is displayed. Any decimal version of this number or the percent calculation is appropriate. 60% or .6

B) (1 pt) 6/10 or 3/5 is shown as the experimental probability and 5/10 or ½ is shown for the theoretical probability and the student mentions that the experimental probability is greater.
C) (1 pt) Student mentions in some way that experimental is what really happens and theoretical is thinking about what could happen. Experimental would be the same as theoretical if you repeated the experiment a larger number of times.</td></tr>
<tr><td>2</td><td>(Partial Credit)

• 2 of the 3 parts are answered correctly & work is shown with some minor error on the other part.
• Only partial points (½ pt) were earned in 1 or 2 of the parts. Examples of this may include, but are not limited to the following:
- A reduction of a fraction is incorrect.
- Student has a misassumption about the true theoretical probability.
- Discussion on the difference isn’t completely valid, but they do have the correct definition of both types of probability.</td></tr>
<tr><td>1</td><td>(Minimal Credit)

• Only 1 of the 3 parts is answered correctly.
• Partial points (½ pt) were earned in 2 or 3 of the 3 parts.</td></tr>
<tr><td>0</td><td>(No credit)

• Student did not attempt the problem.
• Student attempted all 3 parts, but gave 3 incorrect responses.
• Student earned partial points (½ pt) on 1 of the 3 parts.</td></tr>
</table>

BINGO

Label any 1 space "Free Space"

Cut out the square below, shuffle them, and glue them in the top box. You will have one left over!

1:8	1/36	1:1	0:36
1	1:1	3/4	1:3
1:1	1:3	3:1	1/5
1:7	3:2	3:7	3/5

BINGO QUESTIONS

1) When two dice are rolled, what are the odds of getting a sum of 5?

Answer: 1:8

2) When two dice are rolled, what is the probability of getting a sum of 12?

Answer: 1/36

3) When two dice are rolled, what are the odds of getting an even sum?

Answer: 1:1

4) When two dice are rolled, what are the odds of getting a sum greater than 12?

Answer: 0:36

5) When two dice are rolled, what is the probability of getting a sum less than 13?

Answer: 1

6) A spinner has 8 equal parts, what are the odds of getting an even number if the spinner is spun once?

Answer: 1:1

7) A spinner has 8 equal parts, what is the probability of getting a number greater than 2 if the spinner is spun once?

Answer: $\frac{3}{4}$

8) What are the odds that a month picked at random starts with the letter J?

Answer: 1:3

9) If the probability that an event happens is $\frac{1}{2}$,find the odds that it happens?

Answer: 1:1

10) A local video store advertises that 1 out of 4 customers will receive a free box of popcorn when they rent a video. What are the odds of receiving free popcorn?

Answer: 1:3

11) A local video store advertises that 1 out of 4 customers will receive a free box of popcorn when they rent a video. What are the odds against receiving free popcorn?

Answer: 3:1

12) A local video store advertises that 1 out of 4 customers will receive a free box of popcorn when they rent a video. At the end of the day, 15 customers out of 75 had received free popcorn. What is the experimental probability of getting free popcorn?

Answer: 1/5

13) If Michael made 7 free throws out of 8 during the last basketball game, what are the odds that he will NOT make a basket?

Answer: 1:7

14) A bag contains 6 red marbles, 3 blue marbles, and 1 yellow marble. What are the odds of choosing a red marble?

Answer: 3:2

15) A bag contains 6 red marbles, 3 blue marbles, and 1 yellow marble. What are the odds of choosing a blue marble?

Answer: 3:7

16) A bag contains 6 red marbles, 3 blue marbles, and 1 yellow marble. What is the probability of choosing a red marble?

Answer: 3/5

TASK ROTATION

Essential Question:

What is the difference between probability and odds?

Mastery

Describe the difference between odds and probability in a **<u>report</u>** given to casino goers. You need to make up an example for illustrate both probability and odds.

Inform them about the odds and probability of rolling a sum of 7 on two dice. Explain how you calculated the answer, making sure to show each step.

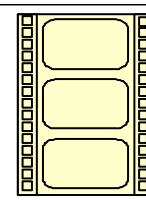
Interpersonal

You and a partner are trying to decide that either probability is easier to understand than odds or odds is easier to understand than probability. Each of you needs to write down you reasons for your preference (you need 3). Now you each get to be Oprah (or Dr. Phil) and interview each other about your preference. Try to have the best interview possible where you reveal the most facts about odds and probability. Each of you writes a reflection piece about how you feel about your preference after the interview. Are your facts still valid or have you changed your mind?

UNDERSTANDING

YOU ARE ASKED BY THE GAMING INDUSTRY TO DEBATE THE USE OF SHOWING PROBABILITY FOR EACH CASINO GAME RATHER THAN ODDS (WHICH THEY CURRENTLY DO). YOUR JOB IS TO CONVINCE THE GENERAL PUBLIC THAT PROBABILITY IS BETTER THAN ODDS WITH AT LEAST 3 SPECIFIC AND VALID EXAMPLES. HOW WOULD THIS EFFECT THE CASINO INDUSTRY AS A WHOLE? WHY DID THEY CHOOSE ODDS VS. PROBABILITY IN THE FIRST PLACE? WRITE OUT YOUR DEBATE.

Self-Expressive



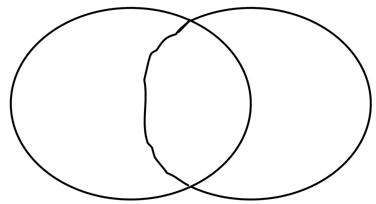
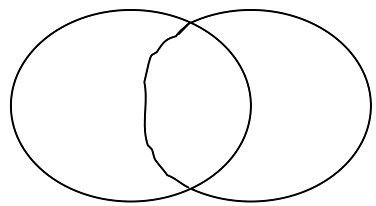
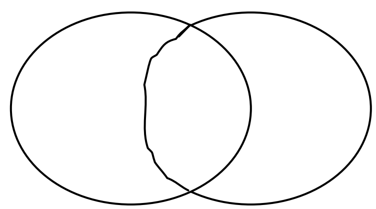
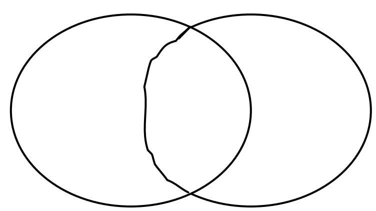
You are going to produce, write, and perform a script for a commercial to the public on why odds are easier to understand in casino games rather than probability. Make sure that you include 3 specific facts about why odds are easier to understand. This is a commercial ordered by the gaming commission, so all your facts must be true – no lies! You will be performing your commercial at the end of the rotation time for the entire class.

Rubric for grading Task Rotation

You must evaluate one person's task rotation, but it cannot be from the same square that you completed!

Mastery	Interpersonal
______ (2 pts) The difference between odds and probability is described in a **<u>report</u>**. ______ (2 pts) An example is made for probability. ______ (2 pts) An example is made for odds. ______ (2 pts) The odds of rolling a sum of 7 on two dice is calculated and each step is shown. ______ (2 pts) The odds of rolling a sum of 7 on two dice is calculated and each step is shown.	_____ (3 pts) Each of you needs to write down you reasons for your preference (you need 3). ______ (5 pts) Interviewed each other about your preference. The best interview possible occurred where you revealed the most facts about odds and probability. ______ (2 pts) Write up on how you feel about your preference after the interview is specific and includes facts.
UNDERSTANDING	Self-Expressive
______ (6 PTS) THREE SPECIFIC AND VALID REASONS ARE WRITTEN OUT ABOUT WHY PROBABILITY IS BETTER THAN ODDS. ______ (2 PTS) THE QUESTION, "HOW WOULD THIS EFFECT THE CASINO INDUSTRY AS A WHOLE?" IS ADDRESSED AND ANSWERED WITH A VALID ARGUMENT. ______ (2 PTS) THE QUESTION, "WHY DID THEY CHOOSE ODDS VS. PROBABILITY IN THE FIRST PLACE?" IS ADDRESSED AND ANSWERED WITH A VALID ARGUMENT.	_____ (3 pts) A script is written for a commercial to the public on why odds are easier to understand in casino games rather than probability. _____ (3 pts) Three specific facts about why odds are easier to understand are included in the script. All the facts must be true! _____ (4 pts) The script is performed in front of the class.

Vocabulary Matrix on Odds and Builder Words

For this assignment, you need to fill out a total of 14 boxes. Feel free to expand any of the boxes on another sheet of paper.				
	Define BOTH in your own words:	Compare and Contrast the two words. (remember, what they have in common goes on the inside of the circle)	Write your own example of a problem for each of the words.	Create a character to represent each word – name them, add their clothes, tell a little about their personality.
Probability vs. Odds				
Independent vs. Dependent				
With Replacement vs. Without Replacement				
Permutation vs. Combination				

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Vocabulary Matrix on Odds and Builder Words

Experimental Probability vs. Theoretical Probability